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RESEARCH ARTICLE

Optimum antenna configuration in MIMO systems: a differential evolution based approach

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ABSTRACT

In this paper, we address the problem of determining the optimum antenna configuration for a multi-input multi-output (MIMO) system at any given signal-to-noise ratio (SNR). We used two-level differential evolution (DE) algorithm that finds both an appropriate expression among a set of candidate expressions within the list of the optimization software used, and the parameter values (*coefficients*) belonging to the selected expression. The results of the proposed expression are compared with the results of high SNR approximation, asymptotic approach and optimum antenna number ratios. It is shown that the numerical outcomes produced by the new expression exhibit very good agreement with the optimum antenna number ratios, and this agreement is almost independent of the specific value of SNR. Copyright © 2010 John Wiley & Sons, Ltd.

KEYWORDS

wireless communications; multi-input multi-output (MIMO) systems; optimization of antenna configuration; differential evolution algorithm

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1. INTRODUCTION

Multi-input multi-output (MIMO) systems are recent developments in wireless communications. They have some novel features that they outperform conventional single-input single-output (SISO) communication systems [1–7]. The SISO system needs high power devices or high order modulations for achieving high data rates. It has been shown that MIMO systems with multiple antennas at both the transmitter and the receiver can improve the wireless link performance in a rich scattering environment and provide increased capacity without the need for additional power or bandwidth requirement [8–14].

Most published papers in the field of antenna selection deal with how to appropriately select the number of antennas [15–17]. Recently, the issue of selecting the number of antennas at the base station (*transmitter*) and at the mobile (*receiver*) to optimize ergodic capacity of a MIMO system was investigated under the assumption that the costs of implementing antennas at the base station and at the mobile are unequal [18]. In that paper, the total system capacity was defined as a linear combination of the uplink and downlink ergodic capacity. The condition required for the optimum antenna number ratio (*optimum antenna configuration*) was derived. How the ratio of the number of antennas at the base station to the number of antennas at the mobile change with signal-to-noise ratio (SNR) and cost ratio, when the total system capacity was maximized was also numerically studied. However, despite its significance, the optimum antenna number ratio analysis presented in Reference [18] was based on the high SNR approximation which leads to poor accuracy at low SNRs. In a recent work by the author [19], a novel expression to determine the optimum antenna number ratios is introduced with the help of differential evolution. Although the results obtained in Reference [19] exhibit very good agreement with the optimum antenna number ratios that maximize the noise-limited asymptotic capacity expression, the accuracy of the expression is not satisfactory when high values of SNR are considered. Therefore, developing a general expression for the determination of optimum antenna number ratios at any given SNR is required.

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Differential evolution (DE) algorithm is a simple and effective evolutionary algorithm that can be used for solving real valued optimization problems [20,21]. The major benefits of the DE algorithm are its simplicity, ease of use, and robustness. Moreover, the DE algorithm uses only a few control parameters and these remain fixed throughout the entire optimization procedure. The DE algorithm has been rapidly gaining in wide acceptance and has already been used in different areas of wireless communications such as prediction of the rain attenuation in millimeter wave radio propagation [22], synthesis of uniform amplitude thinned phased linear arrays [23], sidelobe level reduction on a planar array [24], filter design problems [25–28], synthesis of coplanar strip lines [29], optimization of the difference patterns for monopulse antennas [30], design of optimum gain pyramidal horn antennas, [31] and pattern synthesis [32].

In this paper, we focus on the problem of determining the optimum antenna configuration that optimizes the ergodic capacity in a MIMO system [33]. It is found that the results obtained by the proposed expression exhibits very good agreement with the optimum antenna number ratios that maximize the noise-limited asymptotic capacity. As compared with the previous work by Develi [19], the core contribution of the present paper is offering a general expression that can be used with high accuracy regardless of the value of the SNR. This paper is organized as follows: in the next section, a brief description of MIMO systems is presented by concentrating on the system capacity definitions. Section 3 introduces the application of differential evolution to the problem of determining the optimum antenna configuration. Some numerical results are given in Section 4 to illustrate the accuracy of the proposed expression while Section 5 is a conclusion.

2. MIMO SYSTEM MODEL

In a typical mobile radio propagation environment, the signal transmitted from a transmitter to a receiver is usually corrupted by multipath propagation due to reflections, diffractions and scattering. Before the introduction of MIMO concept, conventional wireless communication systems treated the multipath propagation as a major problem that should be mitigated. As opposed to conventional wireless communication systems, multipath propagation is an essential necessity for the operation of MIMO systems. Block diagram of a MIMO system model is shown in Figure 1. As shown, in MIMO wireless communication systems, n_t antennas (*BS antenna elements*) and n_r antennas (*MS antenna elements*) are employed at the transmitter and the receiver, respectively.

The input–output relation of the downlink channel can be expressed as

$$\mathbf{y} = \sqrt{\rho / n_t}\mathbf{H}\mathbf{x} + \mathbf{v} \tag{1}$$

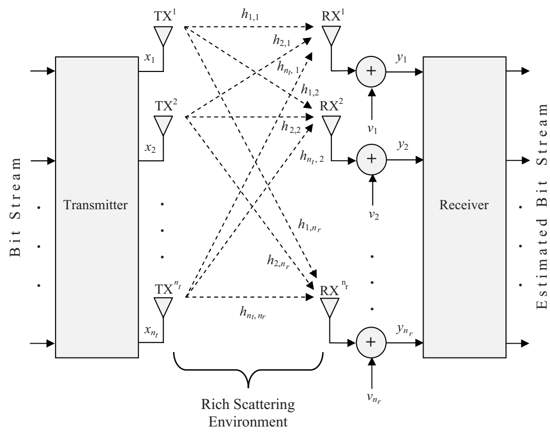

Figure 1. Block diagram of a MIMO system with n_t transmit and n_r receive antennas.

where $\mathrm{y} = [y_1, y_2, ..., y_{n_\mathrm{r}}]^\mathrm{T}$ denotes the received signal vector, ρ denotes the total transmit power, $\mathrm{x} = [x_1, x_2, ..., x_{n_\mathrm{t}}]^\mathrm{T}$ is the transmitted signal vector, and $\mathrm{v} = [v_1, v_2, ..., v_{n_\mathrm{r}}]^\mathrm{T}$ is an additive complex Gaussian noise vector having independent and identically distributed (i.i.d.) elements with zero mean and variance σ^2. **H** is $(n_\mathrm{r} \times n_\mathrm{t})$-dimensional channel matrix given as

$$\mathrm{H} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,n_\mathrm{t}} \\ \vdots & \ddots & \vdots \\ h_{n_\mathrm{r},1} & \cdots & h_{n_\mathrm{r},n_\mathrm{t}} \end{bmatrix} \tag{2}$$

where $h_{i,j}$ describes the transmission characteristics between the ith receive and jth transmit antenna. The elements, $h_{i,j}$, are assumed to be i.i.d. complex Gaussian variables with zero mean and unit variance. We assume that the channel is unknown at the transmitter and perfectly known at the receiver. The mutual information of the MIMO system is given by [16]

$$I = \log_2 \det\left(\mathbf{I}_{n_\mathrm{r}} + \frac{\mathrm{SNR}}{n_\mathrm{t}}\mathbf{H}\,\mathbf{H}^H\right) \text{ bps/Hz} \tag{3}$$

where $\mathbf{I}_{n_\mathrm{r}}$ is the identity matrix, $(\cdot)^H$ is the Hermitian (conjugate transpose) operator, $\mathrm{SNR} \triangleq \rho/\sigma^2$.

The ergodic capacity can be given as

$$C = E\{I\} = E\left\{\log_2 \det\left(\mathbf{I}_{n_\mathrm{r}} + \frac{\mathrm{SNR}}{n_\mathrm{t}}\mathbf{H}\,\mathbf{H}^H\right)\right\} \tag{4}$$

where $E(\cdot)$ denotes the expectation operator. The total system capacity, C_T, in which the n_t and n_r are selected so as to maximize C_T is given by [18]

$$C_\mathrm{T}(n_\mathrm{t}, n_\mathrm{r}, \mathrm{SNR}_1, \mathrm{SNR}_2) = C^{\mathrm{up}}(n_\mathrm{t}, n_\mathrm{r}, \mathrm{SNR}_1) + \lambda C^{\mathrm{down}}(n_\mathrm{t}, n_\mathrm{r}, \mathrm{SNR}_2) \tag{5}$$

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where C^{up} denotes the uplink ergodic capacity and C^{down} denotes the downlink ergodic capacity. SNR_1 and SNR_2 represent the SNRs for the uplink and downlink of the system, respectively. Based on the assumption in Reference [18], $\mathrm{SNR}_1 = \mathrm{SNR}_2 = \mathrm{SNR}$ is considered in this paper. $\lambda > 0$ is a scalar to weight the importance of uplink and downlink capacity. For the total system capacity definition, the following constraints are used

$$n_\mathrm{t}, n_\mathrm{r} > 0, \quad n_\mathrm{t}\mu_\mathrm{t} + n_\mathrm{r}\mu_\mathrm{r} \le 1 \quad \text{and} \quad R = (\mu_\mathrm{r}/\mu_\mathrm{t}) \ge 1 \tag{6}$$

where μ_t and μ_r are the cost per antenna at transmitter and receiver, respectively. R is the cost ratio. In particular, in this paper, it is assumed that the total system capacity is just the sum of the downlink and uplink capacity ($\lambda = 1$). More discussion on the constraints is available in Reference [18].

Because the exact expression for the capacity of the system with n_t transmit and n_r receive antennas reported in Reference [11] is relatively complicated to compute, asymptotic capacity definition was used as an approximation for the optimization in References [34,35]. The noise-limited asymptotic capacity per receive antenna for a system with N_t transmit and N_r receive antennas can be expressed as:

$$\begin{aligned} C(\xi, \mathrm{SNR}) = {} & \log_2\left[1 + \mathrm{SNR} - F\left(\xi, \frac{\mathrm{SNR}}{\xi}\right)\right] \\ & + \xi \log_2\left[1 + \frac{\mathrm{SNR}}{\xi} - F\left(\xi, \frac{\mathrm{SNR}}{\xi}\right)\right] \\ & - \xi \frac{\log_2(e)}{\mathrm{SNR}} F\left(\xi, \frac{\mathrm{SNR}}{\xi}\right) \end{aligned} \tag{7}$$

where $\xi \triangleq N_\mathrm{t}/N_\mathrm{r}$,

$$F(u, m) \triangleq \frac{1}{4}\left[\sqrt{m(1+\sqrt{u})^2 + 1} - \sqrt{m(1-\sqrt{u})^2 + 1}\right]^2$$

The asymptotic capacity yields an extremely accurate approximation to the ergodic capacity even when the number of antennas is very small. For high SNRs, Equation (7) can be approximated as [15]

$$C(\xi, \mathrm{SNR}) \approx \begin{cases} \log_2 \frac{\mathrm{SNR}}{e} - (\xi - 1)\log_2\left(1 - \frac{1}{\xi}\right), & \xi \ge 1 \\ \xi \log_2 \frac{\mathrm{SNR}}{\xi e} - (1-\xi)\log_2(1-\xi), & \xi \le 1 \end{cases} \tag{8}$$

According to the definitions given by Equations (7) and (8), Equation (5) can be obtained as [18]

$$\begin{aligned} C_\mathrm{T}(n_\mathrm{t}, n_\mathrm{r}, \mathrm{SNR}_1, \mathrm{SNR}_2) = {} & n_\mathrm{t} C\left(\frac{n_\mathrm{r}}{n_\mathrm{t}}, \mathrm{SNR}_1\right) \\ & + \lambda n_\mathrm{r} C\left(\frac{n_\mathrm{t}}{n_\mathrm{r}}, \mathrm{SNR}_2\right) \end{aligned} \tag{9}$$

The optimization of total system capacity is equivalent to the maximization of the function given below [18]:

$$\begin{aligned} \Omega(c_\mathrm{t}, c_\mathrm{r}, \mathrm{SNR}_1, \mathrm{SNR}_2) &= \mu_\mathrm{r} C_\mathrm{T}(n_\mathrm{t}, n_\mathrm{r}, \mathrm{SNR}_1, \mathrm{SNR}_2) \\ &= c_\mathrm{t} C\left[c_\mathrm{r}/(Rc_\mathrm{t}), \mathrm{SNR}_1\right] \\ &\quad + \lambda c_\mathrm{r} C\left(Rc_\mathrm{t}/c_\mathrm{r}, \mathrm{SNR}_2\right)/R \end{aligned} \tag{10}$$

where $c_\mathrm{t} \triangleq n_\mathrm{t}\mu_\mathrm{t}$ and $c_\mathrm{r} \triangleq n_\mathrm{r}\mu_\mathrm{r}$. The following constraints are considered:

$$c_\mathrm{r} + c_\mathrm{t} \le 1, \quad c_\mathrm{r} > 0 \quad \text{and} \quad c_\mathrm{t} > 0$$

The antenna number ratio is defined as $K \triangleq n_\mathrm{t}/n_\mathrm{r} = c_\mathrm{t}R/(1 - c_\mathrm{t})$. Based on the detailed optimization process about maximizing $\Omega(c_\mathrm{t}, c_\mathrm{r}, \mathrm{SNR}_1, \mathrm{SNR}_2)$, the antenna number ratio with maximum total system capacity satisfies the following equality

$$K_\mathrm{opt} e^{R\lambda/K_\mathrm{opt}} \left(1 - \frac{1}{K_\mathrm{opt}}\right)^{(1+\lambda)(1+R)} = \frac{e}{\mathrm{SNR}^{1+\lambda}} \tag{11}$$

Because an analytical expression for the solution of Equation (11) is not available, an asymptotic solution for Equation (11) is introduced in Reference [18]. The K that maximizes the linearly combined uplink and downlink capacity is defined as

$$K_\mathrm{opt} \approx \frac{1}{1 - \left(\frac{e^{1-R\lambda}}{\mathrm{SNR}^{1+\lambda}}\right)^{1/(1+R)(1+\lambda)}} \tag{12}$$

It is useful to note that both Equations (11) and (12) were derived assuming high SNR, which leads to poor accuracy at low SNRs. Motivated by this observation, a new expression has been introduced in Reference [19] to obtain more accurate results on the optimum antenna number ratio. Figure 2 shows the antenna number ratio curves ($\tilde{K}_\mathrm{opt}$) obtained by the expression proposed in Reference [19] *versus* SNR for $R = 15$ and $R = 7$. We can observe from Figure 2 that the results carried out by the expression are in good agreement with the K_opt values at low values of SNR. K_opt denotes the optimum value of K that maximize the noise-limited asymptotic capacity and obtained from the discrete optimization of the function by using brute-force method. Unfortunately, in contrast to the results obtained for low values of SNR, the antenna number ratios carried out by the above mentioned expression are not satisfactory when high values of SNR are considered. Therefore, as compared with the work reported in the literature [19], the aim of this paper is to find an expression that not only suitable for low SNRs but also applicable for high values of SNR.

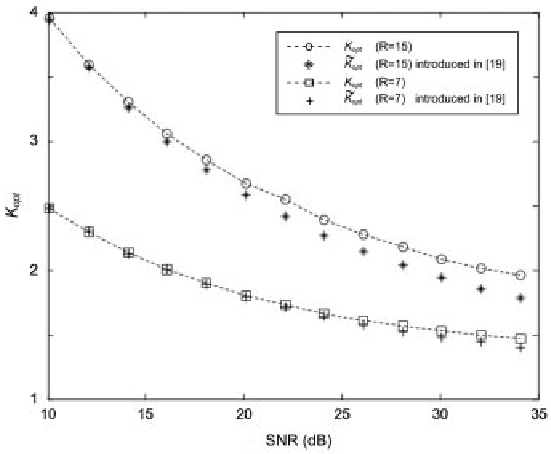

Figure 2. The accuracy of the expression proposed in Reference [19] for different values of SNR when $R = 15$ and $R = 7$.

3. APPLICATION OF DIFFERENTIAL EVOLUTION TO THE PROBLEM

Two-level differential evolution that finds both an appropriate expression and the parameter values (*coefficients*) belonging to the selected expression is used. The software employed consists of a number of available functions from first half to fourth quarter. Also, various optimization algorithms are integrated in the software mentioned. By the help of the DE algorithm, the first step is the process of determining the most appropriate expression among the candidates defined in the list of the optimization software. The expression is essentially derived from a data set. The data set used in this paper contains 294 samples and each sample includes different values of SNR, R and K_{opt}. The respective optimum antenna number ratios, K_{opt}, have been obtained from the noise-limited asymptotic capacity expression given by Equation (7). In these samples, the ranges of system parameters are $1 \leq \text{SNR} \leq 35$(in dB) and $1 \leq R \leq 20$. After a few runs, the following model was offered by the algorithm to represent the input–output relationship between SNR, R and K_p:

$$K_p(\upsilon, \delta) = \frac{b_1 + b_2\upsilon + b_3\delta + b_4\upsilon^2 + b_5\delta^2 + b_6\delta\upsilon}{1 + b_7\upsilon + b_8\delta + b_9\upsilon^2 + b_{10}\delta^2 + b_{11}\delta\upsilon} \quad (13)$$

where $\upsilon = \ln(\text{SNR})$, $\delta = R$, $\{b_1, b_2,\ldots, b_{11}\}$ represent the unknown coefficients that will be determined by differential evolution, and K_p denotes the proposed antenna number ratio. It is useful to note that the formulation of the model is under the control of the DE algorithm, completely. Therefore, it is not possible to direct the formulation process while running the DE algorithm.

The expression in Equation (13) may alternatively be shown in closed form as:

$$K_p = f\,(\text{SNR}, R, b_1, b_2, ..., b_{11}) \quad (14)$$

where $f(\cdot)$ represents the nonlinear relationship between SNR, R and K_p. Given a set of data information, $\{\text{SNR}_k, R_k, K_{\text{opt}}^k\}$, with $k = 1, 2,\ldots, M$, where M is the number of data in a set, the mean absolute model error may be expressed as follows:

$$E = \frac{1}{M}\sum_{k=1}^{M}\left|K_{\text{opt}}^k - K_p^k\right| \quad (15)$$

where K_{opt}^k denotes the optimum value of K that maximize the asymptotic capacity definition given by Equation (7).

By substituting Equation (14) into the above equation, we can rewrite Equation (15) as

$$E = \frac{1}{M}\sum_{k=1}^{M}\left|K_{\text{opt}}^k - f\,(\text{SNR}_k, R_k, b_1, b_2, ..., b_{11})\right| \quad (16)$$

Since SNR_k, R_k and K_{opt}^k are known information, the only unknowns in the above equation are the parameters $\{b_1, b_2,\ldots, b_{11}\}$ of the expression. In this section, the mean absolute model error given in Equation (16) is used as the cost function, and the optimum expression parameters are determined by using the DE algorithm.

As similar to all evolutionary optimization algorithms, the DE algorithm operates on a population with N_{pop} individuals, or candidate solutions [21]. Each individual (or *chromosome*) belonging to the solution vector is composed of N_{par} optimization parameters. In order to establish a starting point for optimum seeking, the population is initialized by randomly generating individuals within the given boundaries:

$$\kappa_{i,j}^p = \kappa_j^{\min} + T_j \times (\kappa_j^{\max} - \kappa_j^{\min}) \quad (17)$$

where $i \in \{1, 2, \cdots, N_{\text{pop}}\}$, $j \in \{1, 2, \cdots, N_{\text{par}}\}$ and $\kappa_{i,j}^p$ denotes the control variable j in individual i at the pth generation. T_j is a random number, uniformly distributed between 0 and 1. $\kappa_j^{\max}$ and $\kappa_j^{\min}$ represent the maximum and minimum permissible values of the jth parameter, respectively. The values of $\kappa_j^{\max}$ and $\kappa_j^{\min}$ are chosen considering the region that probably contain the optimum solution. In order to improve the search efficiency, this region can be scaled by a prior knowledge on the problem to be optimized. It is useful to note that $(\kappa_j^{\max} - \kappa_j^{\min})$ denotes the differential item. After the initialization, the algorithm evolves to the genetic evolution loop by mutation, crossover, and selection operator in sequence. Mutation operation is the key procedure in DE algorithm. The basic idea is to create a difference vector (*mutant vector*) by subtracting two distinct donor vectors randomly selected from the current population, given by

$$\kappa^{N,i} = \kappa^{n,\text{opt}} + P_{\text{mut}}(\kappa^{n,p_1} - \kappa^{n,p_2}), \quad i \neq p_1 \quad \text{and} \quad i \neq p_2. \quad (18)$$

where the superscript N denotes the mating pool. $\kappa^{n,\text{opt}}$ represents the best individual. P_{mut} is the real-valued factor

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commonly with the range [0.1, 1] that scales the differential variations, and therefore controls mutation operation. κ^{n,p_1} and κ^{n,p_2} are the two randomly selected individuals at the nth generation. It should be noted that they are different from each other and also different from $\kappa^{n,\text{opt}}$. Crossover is the second operation in which a trial vector $\kappa^{c,i}$ is formed as follows:

$$(\kappa^{c,i})_j = \begin{cases} (\kappa^{N,i})_j, & \beta \leq P_{\text{cross}} \\ (\kappa^{n,i})_j, & \text{otherwise} \end{cases} \quad (19)$$

where the superscript c denotes children population, β is a real random number in the range of [0,1], and P_{cross} is the probability of a real-valued crossover factor. It is useful to note that N_{pop}, N_{par}, P_{cross}, and P_{mut} are the key parameters that must be set by the user in DE. Proper tuning of these parameters would achieve good tradeoff between the global exploration and the local exploitation so as to increase the convergence speed and efficiency of the search process [21].

The selection step is the final operation in DE algorithm in order to produce better offspring. Each child competes with its parent, and survives only if its fitness is better. The fitness values of the children are computed by using the cost function given by Equation (16). As a result, all the individuals of the next generation are as good as or better than their counterparts in the current generation. Following this, the next round of genetic evolution then begins. These processes are repeated until a termination criterion is satisfied or a predetermined generation number is obtained [19–21].

4. NUMERICAL RESULTS

This section presents some numerical results in order to illustrate the accuracy of the expression developed in the previous section. Since the number of unknown parameters of the expression in Equation (13) is 11, $N_{\text{par}} = 11$. The size of the population N_{pop}, is set to 55 according to the $5 \cdot N_{\text{par}}$ rule mentioned in Reference [20]. The other simulation parameters P_{mut} and P_{cross} are taken as 0.8 and 0.9, respectively. Our choice of the values for the simulation parameters follows the suggestions in References [20,36]. Because the determination of these parameters has a great influence on the result in the DE algorithm, it needs to reasonably determine and set for a better result. The *coefficients* of the expression are then optimally designated for the determination of optimum antenna number ratio so that the output values carried out by the expression converge to the target data. This is achieved by tuning the parameters of the proposed expression by differential evolution so as to minimize the cost function given by Equation (16). The algorithm is terminated when the value of objective function is less than 10^{-4}. The algorithm reached the termination criterion at the 47th iteration. The optimal coefficients are: $b_1 = -2.3018$, $b_2 = -6.9533$, $b_3 = 5.2338$, $b_4 = 2.5372$, $b_5 = 2.0147$, $b_6 = 2.7389$, $b_7 = -6.8401$, $b_8 = 3.9170$, $b_9 = 2.5201$, $b_{10} = 0.0369$, and $b_{11} = 2.6460$.

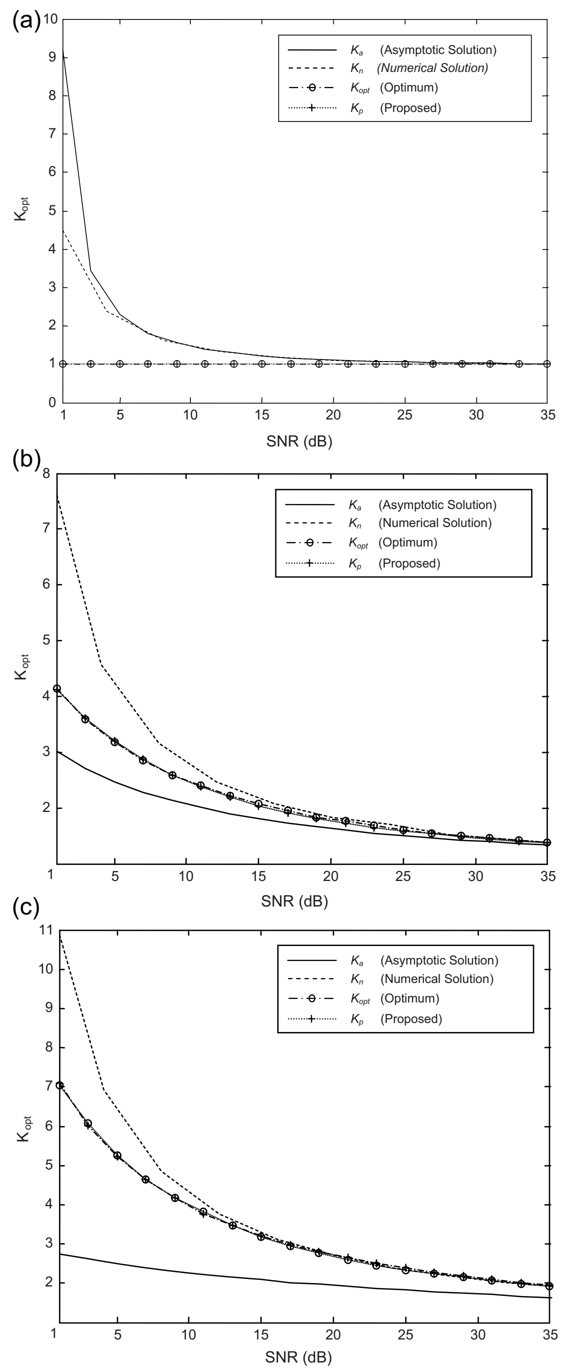

Figure 3. The antenna number ratio curves *versus* SNR for three different values of R; (a) $R = 1$, (b) $R = 7$, and (c) $R = 15$.

Figure 3 shows the antenna number ratio curves as a function of SNR. It is useful to note that the optimum antenna number ratio increases as R increases regardless of the SNR [18,19]. For comparison, the following results are also included in Figure 3: (i) the desired optimum antenna number ratios (K_{opt}) that maximize the noise-limited asymptotic capacity definition given by Equation (7) obtained from discrete optimization of the function by using brute-force method; (ii) the K_n curves obtained by the numerical solution of Equation (11); (iii) the K_a curves computed by Equation (12). As mentioned in Section 2, the results carried out by Equation (12) are the asymptotic solution of Equation (11). It is seen in Figure 3(a), (b), and (c) that the proposed method significantly outperforms the asymptotic solution and the numerical solution. This is a predictable result since both Equations (11) and (12) are good approximations when the SNR goes to higher values. It is also seen that the results carried out by the proposed expression, K_p, provide a very close fit to the K_opt values. Therefore, on the contrary of the approach reported in Reference [19], the results of the new expression are also maximizing the MIMO system capacity at any given SNR.

Finally, we present the computation times needed for the solution of each approach mentioned in this paper. All calculations were executed on a personal computer based on a Pentium IV processor running at 2.4 GHz and equipped with 1 GB of RAM memory. It is found that the computation time needed for solving Equation (7) using brute force method is nearly 5 min while the computation times for numerically solving Equation (11) and solving the expression presented in this paper were both almost 2 s. However, it is useful to note that Equation (11) is a high SNR approximation which leads to poor accuracy at low SNRs. As can be seen, the expression proposed in this paper provides a convenient way for designers to simply determine the optimum antenna number ratios for MIMO systems at any given SNR rather than using brute-force methods that attempt to examine as many as possible solution candidates to find the optimum antenna number ratios.

5. CONCLUSION

It is well known that the differential evolution algorithm is a highly efficient technique for finding optimum effectively with a smaller probability of falling in local optima than other evolutionary algorithms [20]. In this paper, by the help of the DE algorithm, a new expression is proposed to determine the optimum antenna number ratios for MIMO systems at any given value of SNR. The proposed expression is derived by applying function approximation and curve-fitting technique to the respective optimum antenna number ratios that maximize the noise-limited asymptotic capacity definition. The differential evolution is employed in order to properly adjust all necessary coefficients of the presented expression. The accuracy of the expression is verified through numerical comparisons.

The classical exhaustive search methods or brute-force methods that attempt to examine as many as possible solution candidates to find the optimal result are weakest methods since extremely large amount of computer process time are needed to obtain reasonable results. In addition, these methods are appropriate to the problems with small and finite solution population space. Compared with classical exhaustive or brute-force search methods, differential evolution is much more effective and efficient in term of solution time and problem solvability. When the results obtained by Equation (13) are compared with those calculated by the expression introduced in Reference [19], it can be concluded that the main contributions of this paper are showing the use of a bio-inspired DE algorithm to solve an optimization problem and determining the optimum antenna configuration for a MIMO system at any given SNR. As a final remark, using an equal number of antennas at the base station and at the mobile is a tendency to maximize the total system capacity at very high values of the SNR. In this case, the optimum antenna number ratios will equal to 1, and therefore there is no need to refer to the proposed expression.

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